Partial Derivatives

Problem 1. *Is the following function continuous.*

$$f(x,y) = \begin{cases} y\,\sin\left(\frac{1}{x^2+y^2}\right) & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

Find the partial derivatives of f at $(0,0)$.

Problem 2. *Find the partial derivatives at $(0,0)$ of the following function:*

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

Is the function f continuous at $(0,0)$. [Answer: $f_x(0,0) = f_y(0,0) = 0$, but f is not continuous.]

Problem 3 (Stewart, Exercise 14.3.101)**.** *Use Clairaut's Theorem to show that if the third-order partial derivatives of f are continuous, then $f_{xyy} = f_{yxy} = f_{yyx}$.*

Problem 4 (Stewart, Exercise 14.3.103)**.** *If*

$$f(x,y) = x(x^2+y^2)^{-3/2}e^{\sin(x^2y)},$$

find $f_x(1,0)$. [Hint: Can we simplify the function evaluating on certain constant one of the variables?]

Problem 5. *If $f\colon \mathbb{R}^2 \to \mathbb{R}$ is differentiable and constant on the line $x = y$, argue that $f_x(t,t) = -f_y(t,t)$ for all $t \in \mathbb{R}$.*

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.